# Neutral axis in CFS-reinforced RC beams influenced by RC beams' strength

Industrial Technology of Nihon University ○ Ming-Chien HSU
Tetukazu KIDA; Tadashi ABE
Toshiaki SAWANO; Yoshitaka OZAWA

## 1. Introduction

The adhesion of carbon fiber sheet (CFS) offers various advantages such as the construction ability and the reduction of construction time. Therefore, it has been recently found in the increasing applications to the strengthening of RC member and the repair of cracked structural member. This strengthening method has been the subject of a number of studies leading to reports on suitable design methods, mechanisms and its reinforcing effects [1-5].

The present paper deals with the initial concrete strain that effects the neutral axis calculation in the theoretical bending moment capacity for the RC beams reinforced with CFS (hereafter, called CFS-reinforced RC beam) by using the calculation from CEB-FIP [6]. Moreover, the initial concrete strain is calculated from the RC beam without any reinforced with CFS. Once the initial concrete strain is been calculated, the neutral axis of CFS-reinforced RC beams can be calculated from the CFS strain and the initial concrete strain [6]. Therefore, the authors are going to use two different calculation approaches that are the ultimate concrete compression strength and the resisting bending moment for the initial concrete strain. Furthermore, by using these two approaches of neutral axis calculation, the theoretical ultimate flexural bending moment of CFS-reinforced RC beams will be calculated and compared with the experiment results.

Table 1 Physical property of concrete and steel re-bars

| Test specimen | Concrete | Re-bar (SD295A / D16) | | |
|---|---|---|---|---|
| | Compressive strength ($N/mm^2$) | Yield strength ($N/mm^2$) | Tensile strength ($N/mm^2$) | Young's modulus ($kN/mm^2$) |
| RC beam Type A | 38.5 | 368 | 568 | 200 |
| RC beam Type B | 41.5 | | | |

Table 2 Physical properties of CFS

| Name of CFS | Unit weight ($g/m^2$) | Tensile strength ($N/mm^2$) | Young's modulus ($kN/mm^2$) |
|---|---|---|---|
| High strength CFS | 202 | 4420 (Axial)<br>492 (Transversal) | 235 |

## 2. Preparation of experimental specimens [7]

### 1) Material

Ordinary Portland cement and coarse aggregate with a maximum size of 20mm were used for the experimental specimens. The D16 steel re-bars of SD 295A type were used. The physical properties of concrete and steel re-bar are listed in Table 1. The high-strength continuous carbon fiber sheet with a width of 300mm and a thickness of 0.111mm was used, and the physical properties of CFS are listed in Table 2.

### 2) Specimen set-up

Two types of RC beam specimens with different depths were prepared. The specimen sizes and the selected measuring points are shown in Fig. 1. The RC beam specimens had reinforced with CFS is referred to as "CFS-reinforced RC beams," hereafter.

(1) Type A: It had the span of 2000mm, the width of 300mm and the height of 210mm as to the cross-section. There were three steel re-bars on the tension sides with the effective depth of 172mm and two steel re-bars on compression sides.

(2) Type B: It had the span and the width same as Type A, but the height was 250mm. The arrangement of the steel re-bars was same as Type A, but the effective depth was 212mm.

### 3) CFS Bonding Procedures

First, the bottom surface of RC beams was ground smoothly. Then, primer and primary epoxy were applied to the bottom surface of RC beams. A single layer of CFS was then placed between two supports at the bottom of RC beams in the direction of the primary reinforcement by used the epoxy resin as an adhesive agent.

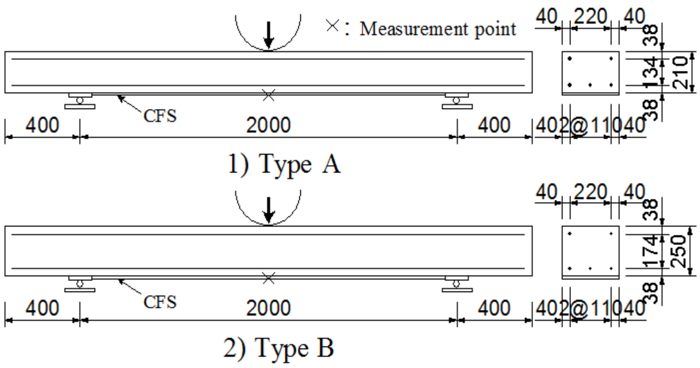


Fig. 1 Specimen size and Steel Re-bars arrangement

CFS 補強が RC はり強度の中立軸に及ぼす影響

徐　銘謙、木田哲量、阿部　忠、澤野利章、小澤善隆

## 3. Outline of experiments [7]

This was a flexural experiment in which a wheel was rest at the center of the span (that was at the point of maximum flexural stress). The load was increased in increments of 5.0kN by the loading controller after each experiment.

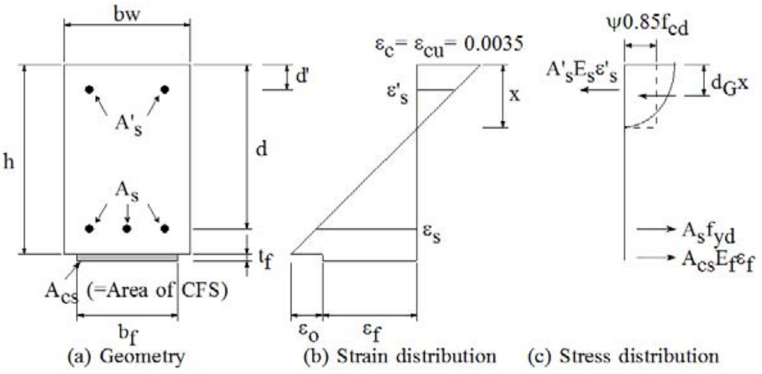


Fig. 2 Cross section of maximum limit state in bending

## 4. Results of experiments [7]

### 1) Failure modes

In the flexural experiments under the static load, all CFS-reinforced RC beams suffered the flexural failure as the load was being increased with cracks developing at an angle of approximately 55-60 degrees from a point directly below the wheel.

For the CFS, it peeled away from the concrete surface due to the tension at the center of the RC beam, with the peeling progressing toward the supports. Therefore, the CFS was not torn on any of the specimens; it was peeled off from the bottom of RC beams after flexural failure.

### 2) Ultimate flexural load-carrying capacity

The ultimate flexural load-carrying capacities were 120.3kN and 137.5kN for Types A and B, respectively. Again, all CFS-reinforced RC beams also failed in flexure under the loading point.

## 5. The theoretical ultimate flexural bending moment

The theoretical ultimate flexural load-carrying capacity, $P_u$, is calculated in Eq. 1. The ultimate flexural bending moment equations are going to be explaining in the flowing for the CEB-FIP [6].

$$P_u = 4 \cdot M_u / L \qquad (1)$$

Where, $M_u$ is the ultimate flexural bending moment; $L$ is the span length.

### 1) Ultimate Flexural Bending Moment Equation from CEB-FIP [6]

The ultimate limit state design of CEB-FIP [6] is based on the critical cross section that occurs by yielding of the tensile reinforcements followed by crushing of concrete. Fig. 2 [6] shows the design bending moment of the strengthened cross section that based on principles of RC design. Eq. 2 is the design ultimate flexural bending moment capacity.

$$\begin{aligned} M_{Rd} = {} & A_s \cdot f_{rd} \cdot (d - \delta_{Gx}) \\ & + A_s' \cdot E_s \cdot \varepsilon_s' \cdot (\delta_{Gx} - d') \\ & + A_{cs} \cdot E_f \cdot \varepsilon_f \cdot (\mathrm{h} - \delta_{Gx}) \end{aligned} \qquad (2)$$

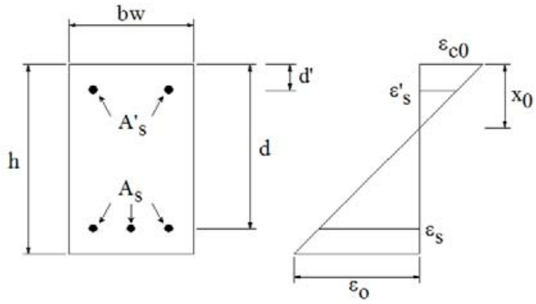


Fig. 3 Initial situation

Where, $x = \dfrac{A_s f_{yd} + A_{cs} E_f \varepsilon_f + A_s' E_s \varepsilon_s'}{0.85 \psi f_{cd} b_w}$,

$\varepsilon_s' = \varepsilon_{cu} \dfrac{d-x}{x} \geq \dfrac{f_{yd}}{E_s}$ ; $\varepsilon_f = \varepsilon_{cu} \dfrac{h-x}{x} - \varepsilon_0 \leq \varepsilon_y$ ,

$f_{yd}$ is the yielding strength of reinforcement, $\sigma_s'$ is the stress of compression reinforcement, $A_s$ is the amount of reinforcement on tension side, $A_s'$ is the amount of reinforcement on compression side, $d$ is the effective depth, $d'$ is the upper thickness of covering concrete, $\varepsilon_f$ is the strain of CFS, $\varepsilon_o$ is initial strain of concrete, $\delta_G$ is stress block centroid coefficient (=0.4), $\psi$ is stress block area coefficient (=0.8), $f_{cd}$ is concrete compression strength, $\varepsilon_{cu}$ is ultimate concrete strength, $b_w$ is the width of RC beam and $x$ is neutral axis depth.

In the Eq. 2, the strain of CFS is been calculated related to the ultimate concrete strain, the high of RC beam, the neutral axis, and the initial strain of concrete. Also, there are two limitations for this equation. The first, the tensile steel re-bars are over its own yield strength. The second, the CFS is not excesses its own ultimate strength. In the other word, the full strain of CFS was not been considered into the calculation since the concrete had been taken the initial strain into the consideration. Moreover, the stress distribution for calculating the neutral axis in the Eq. 2 had

Table 3 Calculation results and experiment results

| | | Initial concrete strain ($\varepsilon_{c0}$) | Netural axle (mm) | CFS strain ($\varepsilon_f$) | Ultimate flexural load-carrying capacity (kN) | | Test / Theoretical |
|---|---|---|---|---|---|---|---|
| | | | | | Test | Theoretical | |
| Ultimate concrete compression strain ($\varepsilon_{ul}$) | Type A | 0.010692 | 36.0 | 0.066206 | 120.9 | 88.8 | 1.36 |
| | | | | | 119.7 | | 1.35 |
| | Type B | 0.010688 | 36.4 | 0.009839 | 139.8 | 123.4 | 1.13 |
| | | | | | 135.1 | | 1.09 |
| Resistance bedning moment ($M_0$) | Type A | 0.002340 | 39.4 | 0.012810 | 120.9 | 107.0 | 1.13 |
| | | | | | 119.7 | | 1.12 |
| | Type B | 0.002392 | 39.6 | 0.016216 | 139.8 | 145.0 | 0.96 |
| | | | | | 135.1 | | 0.93 |

considered the stress of CFS.

### 2) Initial strain of concrete ($\varepsilon_o$)

Initial strain of concrete can be calculated from two different processes that are the ultimate concrete compression strain, and the resisting bending moment. Eq. 3 shows the calculation of initial concrete strain and Fig. 3 [6] shows the initial situation. Furthermore, for calculating the initial strain of concrete for CFS-reinforced RC beams, the ultimate concrete compression strain is going to be the concrete strain at the top of fiber ($\varepsilon_{c0}$) in Eq. 3 and. The resistance bending moment is going to be the cracking moment ($M_0$) in Eq. 3.

$$\varepsilon_o = \varepsilon_{c0}\,(h - x_0)/x_0 \qquad (3)$$

Where, $\varepsilon_{co} = M_0 x_0 / E_c I_{c0}$ , $\varepsilon_{c0}$ is concrete strain at the top fiber, $M_0$ is the cracking moment, $x_0$ is the neutral axis depth for RC beam without CFS, $I_{c0}$ the moment inertia of transformed cracked section [4], $E_c$ is concrete young modulus.

### A) Ultimate concrete compression strain ($\varepsilon_{cu}$)

The ultimate concrete compression strain is set to be 0.0035 [8] and had been used as the initial concrete strain for calculating the ultimate flexural bending moment capacity for CFS-reinforced RC beams. In the other word, the RC beam has been suffered or crashing in certain loading conditions before reinforced with CFS by using this ultimate concrete compression strain.

### B) Resisting bending moment ($M_0$)

The resisting bending moment [9] is been used for calculating the cracking moment in the concrete initial strain. In this calculation, the RC beams has not reinforced by CFS or anything else. Therefore, the neutral axis depth is just for the simple RC beam. Eq. 4 shows the calculation of the resisting bending moment.

For this calculation, the concrete strain ($\varepsilon_{c0}$) at the top of fiber is not close to or greater than the ultimate strain ($\varepsilon_{ul}$). In the other word, the RC beam is not crash condition.

$$M_0 = f_{yd}\left\{\frac{b_w x_0^2\left(d - \frac{x_0}{3}\right) + 2nA_s'(x_0 - d')(d - d')}{2n(d - x_0)}\right\} \quad \dots (4)$$

Where, $n = E_s/E_c$, and $x_0 = -\frac{n(A_s + A_s')}{b_w} + \sqrt{\left\{\frac{n(A_s + A_s')}{b_w}\right\}^2 + \frac{2n}{b_w}(A_s d + A_s' d')}$.

## 6. Neutral axis of CFS-reinforced RC beams

From the Eq. 2, the strain of CFS used in the CFS-reinforced RC beams is calculated from the relationship between ultimate concrete strain and initial strain of concrete, and between the height of RC beams and the neutral axis. Moreover, the neutral axis calculation is effect by the initial concrete strain and the CFS strain that can be found in the Fig. 2. Therefore, by using two different approach of the initial strain, the neutral axis calculated from ultimate concrete compression strain is less than the one calculated from the resisting bending moment that shows on Table 3. It's because the RC beams are in the crashing condition before reinforced with CFS by using the ultimate concrete compression strain to calculate the initial strain of concrete in the CFS-reinforced RC beam calculation.

As the results for the initial strain calculated from the ultimate concrete compression strain, the height of neutral axis for CFS-reinforced RC beam is 36.0mm and 36.4mm for Type A and B, respectively. Moreover, the compression steel re-bars are under the tensile condition since the clearance is 38mm. As the results for the initial strain calculated from the resistance bending moment, the height of neutral axis is 39.4mm and 39.6mm for Type A and B, respectively. Moreover, the compression steel re-bars are under the compression condition. Furthermore, the initial concrete strength calculated from the ultimate concrete compression strain is larger than calculated from the resistance bending moment.

## 7. Experimental and theatrical results

Table 3 shows the results of the calculation. By using the ultimate compression strain, the results are 88.8kN and 123.4kN for Type A and B, respectively. By using the resistance bending moment, the results are 107.0kN and 145.0 for Type A and B, respectively.

By comparing with the experimental results, the calculations used the ultimate concrete compression strain are 36% and 11% less for Type A and B, respectively. The calculations used the resistance bending moment are 13% less and 6% over for Type A and B, respectively.

## 8. Conclusion

1) As results for the ultimate flexural load -carrying capacity, the initial concrete strain of CFS-reinforced RC beams calculated from the ultimate concrete compression strain is less than the one calculated from the resistance bending moment. It's because the RC beams are under crashing condition before RC beams reinforced with CFS by calculated from the ultimate concrete compression strain.
2) As the results for the initial strain calculated from the ultimate concrete compression strain, the neutral axis height for CFS-reinforced RC beam is less than the clearance of 38mm as the compression steel re-bars are under the tensile condition. As the results for the initial strain calculated from the resistance bending moment, the neutral axis height is greater than the clearance of 38mm as the compression steel re-bars are under the compression condition.
3) The neutral axis of CFS-reinforced RC beams is influenced by the initial strain of concrete that before reinforced with CFS. It's because this initial concrete strain will lead the calculation to calculate the strain of CFS in the CFS-reinforced RC beams. Moreover, the strain of CFS should not excess its own ultimate strain.
4) The initial strain of concrete for calculating the ultimate flexural bending moment capacity of CFS-reinforced RC beams will depend on the condition of the RC beams. It's because if the RC beams are crashing or badly damage, the ultimate concrete strain can be used as the initial strain of concrete. Moreover, if the RC beams have no or not much damage, the resisting bending moment can be used to calculate the initial strain of concrete.

## 9. Acknowledgement

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## 10. Reference

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