Optimal Shape and Topology of Structure Searched by Ants’ Foraging Behavior

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Abstract

This paper presents an effective method for designing structures using cellular automata, representing a simple conceptual basis for the self-organization of structural systems. The method is sufficiently simple to solve topology optimization problems as pure 0-1 problems, and yet sufficiently complex to express a wide variety of complicated topologies. A local rule about birth and death of cells, that is a new idea from pheromone’s properties of ants, is introduced in order to search for solutions. The local rule has binary characteristic, spatial summation characteristic, and temporal summation characteristic of inputs. Because of the temporal summation characteristic of inputs, especially the method is able to apply to problems of structure subjected to periodic forces or periodic support conditions. The effectiveness of the present method is demonstrated through numerical examples of the typical topology optimization problem.

Keyword: Optimization, Structure, Shape, Topology, Heuristics

1. Introduction

The structural optimization problem can be classified into two different sub-problems, namely shape optimization problem and topology optimization problem. The shape and topology of a structure are defined by a set of design variables, and these design variables are adjusted in order to achieve some objectives, such as minimum volume. Such optimization problems can be solved iteratively, using gradient-based techniques. Introducing more design variables increases the complexity of the optimization problem. Therefore, it becomes difficult to solve the optimization problem by using gradient-based MP techniques.

Heuristics like genetic algorithms as an example overcomes such difficulties associated with traditional techniques. Genetic algorithms are based on the random mutation and natural selection. On the other hand, self-organization is also considered to be one of the most important mechanisms of evolution. In addition to natural selection, self-organization

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plays an important role in generating the detailed structure of life.

Some species of ants can discover the shortest path from their nest to food source as shown in Fig. 1. It has already become clear that the chemical called pheromone plays important role for the search. The pheromone has two characteristics. One is accumulation, and the other is evaporation. Accumulation of the pheromone acts on learning and evaporation acts on forgetting. These two characteristics are also important for searching the optimal solution. The method to be presented here is a heuristic method of using such accumulation and evaporation, namely, learning and forgetting, for structural optimization. This is a kind of cellular automata. This is a simple concept of self-organization of structural systems. The proposed methods applying cellular automaton theory are sufficiently simple to solve the topology optimization problem just as pure 0-1 problems, but sufficiently complex to express a wide variety of complicated topology. A local rule about birth and death of cells, that is a new idea from pheromone’s properties of ants, is given in this method. The proposed method in this paper offers a new approach to structural optimization, and overcomes most of the problems associated with traditional techniques.

2. Algorithm

First a design domain is divided into regular lattice of square cells, which are identical to the finite elements. A piece of material is then given in the design domain as an initial design consisting of connecting elements that transfer loads to the supports, as shown in Fig. 2. A stress analysis is carried out to determine the response of the structure using a finite element method. The stress level at each point can be measured by the von Mises stress, which is one of the most frequently used criteria for isotropic materials. For plane stress problems, the von Mises stress is defined by the following equation:

$$\sigma = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3 \tau_{xy}^2},$$

(1)

where \(\sigma_x\) and \(\sigma_y\) are the normal stresses in x and y directions, respectively, and \(\tau_{xy}\) is the shear stress.

The procedure is summarized as follows:

Step 1. Divide design domain into regular lattice of square cells.

Step 2. Give an arbitrary initial design so that load is transferred to the supports.

Step 3. Carry out a stress analysis using a Finite Element Analysis.

Step 4. Update the existence of the material elements based on the local rule and measured stress.

Step 5. Repeat steps 3 and 4.

3. Local Rule

In the evolution of cellular automaton, the value of the center cell is updated according to a rule that is de-
pendent on the values of cells in the surrounding neighborhood. In this study, the neighborhood of a specified cell is defined as the cell itself and the four cells immediately adjacent to it, as shown in Fig. 3.

\[ u_{k+1} = (1-\lambda) u_k + \omega (\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4), (0 < \lambda < 1), \]  

(2)

\[ \chi = \begin{cases} +1 & (\sigma > \sigma^E) \\ 0 & (\sigma \leq \sigma^E) \end{cases} \]  

(3)

An expected value \( \sigma^E \) of the von Mises stress is introduced to define a cellular automaton rule. When the von Mises stress of a cell in the neighborhood is over the expected value \( \sigma^E \), it is considered that the cell sends input signal to the target cell as expressed in equation (3). On the other hand, when the stress is not over the expected value, the cell sends no input signal. This is the binary characteristic of inputs. The inputs accumulate on each cell, and we call the accumulated quantity a potential. As the result, the sum total of input signal is added to the potential of the target cell as expressed by the second term in equation (2). This is the spatial summation characteristic of inputs. In the equation, \( u_k \) and \( u_{k+1} \) are the potential at discrete time \( k \) and \( k+1 \), respectively. The primary term of the equation means that the potential is decreasing with time. This is the temporal summation characteristic of inputs. Therefore, if there are some inputs, the potential will increase with time. Increase of the potential acts on learning. If there is no input, the potential will decrease with time. Decrease of the potential acts on forgetting. We can see that the potential value increases, while some inputs are continuing, and discontinuation of inputs begins to decrease the potential value as shown in Fig. 4. When the difference of potential and threshold \( u - \varepsilon \) is positive, a material element will be added to the target cell of the present structure. On the other hand, when the potential is less than the threshold, a material element will be removed from the present structure as shown in Fig. 5. This is the nonlinear threshold function.
Characteristics of the local rule conceived from properties of pheromone are summarized as follows:
1) Binary characteristic of inputs
2) Spatial summation characteristics of inputs
3) Temporal summation characteristics of inputs
4) Nonlinear threshold function

4. Minimum Weight Problem

First, we consider a well-known structural optimization problem; a two-bar frame subjected to a single load. The rectangular design domain is 1,000mm×2,400mm, as shown in Fig. 6, and is divided into 25×60 four-nodes plane stress elements of equal size. The thickness of the plate is 10mm. Young’s modulus E = 100 GPa and Poisson’s ratio \( \nu = 0.3 \) are assumed. A vertical load of P = 800 N is applied at middle of the free end. The initial design is composed of connecting elements that transfer loads to the supports, as shown in (a) of Figs. 8 and 9. In this problem, the expected value \( \sigma^E \) of von Mises is specified as 0.25 MPa. Figs. 8 and 9 show stages of evolution started from different initial design, respectively. The stress concentration on the left end of the beam can be seen in (a). The beam bifurcates into a two-bar frame after a certain number of iterations. The final result, shown in (f), is similar to the two-bar frame that can be derived analytically. The iteration histories of the maximum, minimum and average von Mises stress are plotted in Fig. 7. The coefficients \( \lambda, \omega \) in equation (2) and the threshold \( e \) are selected as 0.1, 0.2 and 1, respectively.

Next, we consider a Michell type structure with two fixed supports. The rectangular design domain is 10,000mm×5,000mm, as shown in Fig. 10. The design domain is divided into 50×25 four-nodes plane stress elements of equal size. The thickness of the plate is 100mm. Young’s modulus E = 100 GPa and Poisson’s ratio \( \nu = 0.3 \) are assumed. The two corners of the bottom are fixed. A vertical load of F = 1000 N is applied at the middle of lower span. The initial design is composed of connecting elements that transfer loads to the supports, as shown in Fig. 11(a). In this problem, the expected value \( \sigma^E \) of von Mises is specified as 8KPa. Fig. 11 shows selected stages of evolution. The final result is shown in (c).
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Fig. 8 Evolving towards the two-bar frame structure

Fig. 9 Evolving towards the two-bar frame structure

Fig. 10 Design domain for Michell type structure

Fig. 11 Stages of evolution
5. Displacement Constraint Problem

The cantilever beam shown in Fig. 12 is considered as an example of the optimization of structures with displacement constraints. The structure is assumed to be under plane stress conditions. The rectangular design domain is 160mm×100mm, as shown in Fig. 12. The design domain is divided into 80×50 four-nodes plane stress elements of equal size. The thickness of the plate is 1mm. Young’s modulus $E = 207$ GPa and Poisson’s ratio $\nu = 0.3$ are assumed. The left-hand side of the beam is fixed. A vertical load of $P = 3,000$ N is applied at middle of the free end. A displacement constraint is imposed on the vertical displacement at the loading point. The limit for the displacement is 1mm. The limit for the volume of the structure is 5,000 mm$^3$. The initial design is composed of connecting elements that transfer the load to the supports, as shown in Fig. 13(a).

In this case, the mean compliance $C$ given by equation (4) is substituted for von Mises stress as a driving parameter. The mean compliance $C$ of the structure is defined by

$$C = \frac{1}{2} \{P\}^T\{u\}, \quad (4)$$

where $\{P\}$ is the nodal load vector and $\{u\}$ is the nodal displacement vector. The change in the mean compliance as a result of removing the $i$-th element is given by

$$\alpha_i = \frac{1}{2} \{u^i\}[k^i]\{u^i\} \quad (5)$$

where $\{u^i\}$ and $[k^i]$ are the displacement vector and the stiffness matrix of the $i$-th element, respectively. The change in mean compliance $\alpha_i$ is also known as the sensitivity number for the problem. In the case of displacement constraint problem, the sensitivity number $\alpha_i$ takes the place of $\alpha_i$ in equation (3). In this example, the expected value of the sensitivity number $\alpha^E$ is specified as 1.2 Nmm. Fig. 13 shows selected stages of evolution. The final result is shown in (c). The histories of the volume of the structure and the vertical displacement at the load point are plotted in Fig. 14.

Another example named MBB beam problem is shown in Fig. 16. The rectangular design domain is 2,400mm×400mm. The design domain is divided into
20×120 fours-node plane stress elements of equal size. The thickness of the plate is 1mm. Young’s modulus $E = 200$ GPa and Poisson’s ratio $\nu = 0.3$ are assumed. The two corners of the bottom are fixed. A vertical load of $P = 20,000$ N is applied at the middle of upper span. A displacement constraint is imposed on the vertical displacement at the loading point. The limit for the displacement is 10mm. The limit for the volume of the structure is 500,000 mm$^3$. The initial design is composed of connecting elements that transfer the load to the supports, as shown in Fig. 17(a). The final result is shown in Fig. 17(c).

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Fig. 14 Volume and vertical displacement histories

Fig. 15 Histories of mean compliances

Fig. 16 Design Domain and structural conditions for the MBB beam

(a) Initial design

(b)

(c) Final result

Fig. 17 Stages of evolution
6. Optimization Against Periodic Force

Now, we consider minimum weight problem of structure subjected to periodic forces. The proposed method for optimization has temporal summation characteristics of inputs as mentioned above. So the proposed method is able to apply to problems of structure subjected to periodic forces. The coefficients in equation (2) are replaced in order to be related with period of the external force as follows:

\[ u_{e+1} = (1 - \lambda \Delta t) u_k + \omega \Delta t (X_0 + X_1 + X_2 + X_3 + X_4), \quad (6) \]

where \( \Delta t \) is time interval, \( \lambda \) and \( \omega \) are coefficients calculated from the period of loading by using equation (7) and (8), respectively. In these equations, \( c \) and \( e \) means a saturation of the potential and a threshold, respectively. If a period \( T \) is given in a problem, we can determine \( \lambda \) by equation (7). Then, we can determine \( \omega \) by equation (8). In order to show effectiveness of the method, we consider a structural optimization problem; a two-bar frame subjected to a single load mentioned above, but the direction of load varies periodically as shown in Fig. 18. Threshold \( \epsilon = 1 \), saturation \( u_s = 2 \) and time interval \( \Delta t = 0.1 \) are assumed. When the period is 10 sec, appropriate values of the coefficients \( \lambda \) and \( \omega \) are 0.0693 and 0.139, respectively. Fig. 19 shows the stages of evolution.

\[ \lambda = -\frac{\log \frac{\epsilon}{u_s}}{T} \quad (7) \]

\[ \omega = u_s \lambda \quad (8) \]

Fig. 18 Stages of evolution

Fig. 19 Structure under Periodic Force
7. Conclusions

In this paper, an effective method for designing structures using a cellular automaton was presented. This provides a simple conceptual basis for the self-organization of structural systems. The effectiveness of the method was demonstrated through numerical example of the typical topology optimization problem. Moreover, the proposed method can be applied to also shape optimization of structures under periodic forces.

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References

アリの探索行動が見付けた構造の最適形状と最適位相

概要

構造の形状決定問題は、大きさ、形状、トポロジーの3つの最適化問題に分類できる。大きさを決定する問題は、比較的少ない設計変数で問題を構成することができる。形状やトポロジーを決定する問題では、複雑な形状や種々のトポロジーを考慮するために設計変数の増加を避けることができない。これらの設計変数は、目的関数と制約条件を設定することによって最適化問題として定式化され、勾配を用いた数理計画法によって解くことが可能である。しかし、設計変数の増加にともなって問題の複雑さも増加し、勾配ベースの数理計画手法を使って解くことは難しくなる。

数理計画法に起因するこのような困難は、例えば、遺伝的アルゴリズムや免疫アルゴリズム、シミュレーティッドアニーリングのような発見的手法を応用することによって克服することができる。著者は、セルの出現と消滅に関する単純な局所規則に触のフェロモンの性質に着想を得た発見的手法を提案した。応力の目標値を超えるセルが近傍にあった場合、目標セルにはその近傍セルより一定の入力が与えられ、この入力が目標セルのポテンシャルを増加させる逆入力がない場合はポテンシャルが減衰すると考える。そして、ポテンシャルがある閾値を超えると目標セルに材料が出現するというものである。

本論では、この手法を典型的な構造最適化問題である最小重量問題に適用して、その有効性を示す。さらに、パラメータの値と時間の関係を明確にし、周期的に変動する荷重条件下の構造最適化問題へ適用して、これらの問題に対する提案する手法の適用性と有効性を示す。この方法では、セルの出現と消滅に関する局所的な規則だけが必要であり、設計感度などを必要としない。このため、本手法は前述の数理計画手法を用いた方法に起因する困難の大部分を克服し、さらに周期的に変動する条件下の構造最適化問題への新しいアプローチを提供する。
Biographical Sketches of the Author

Kazuo Mitsui received his Bachelor’s Degree of Engineering in 1977, his Master’s Degree of Engineering in 1979, and Doctor’s Degree of Engineering in 1995 from Nihon University. Dr. Mitsui is an associate professor of the department of Mathematical Engineering at College of Industrial Technology, Nihon University. He is a member of The International Association for Shell and Spatial Structures (IASS), Architectural Institute of Japan (AIJ), The Japan Society of Mechanical Engineers (JSME), The Japan Society for Computational Engineering and Science (JSCES)