

# Deformation of RC Slabs under the Static-Load

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## 1. Introduction

Experimental researches of reinforced concrete (RC) members have been carried out over the years at the College of Industrial Technology of Nihon University. Experimental researches like the static-load test applied on RC slabs had been done. For the further studies, Finite Element Method (FEM) would be one of the choices to help authors to understand more on the experimental results of the RC slabs that had been done for the past. The FEM program used in this research is called DIANA<sup>[1]</sup>.

The authors adopted the parameters of RC slab's properties from the standard specifications for concrete structures<sup>[2]</sup>, CEB-FIP Mode Code 90<sup>[3]</sup>, ACI Code, and some theories that had proposed before by others. Further, the experimental results and the FEM results would be compared to each other. Also, a coefficient of parameter, related to ultimate strain of concrete's tension stiffness, would be found to match the experimental results with the FEM results.

## 2. Test Materials, Specimen Sizes, and Test Method<sup>[4 and 5]</sup>

### A. Experimental Material

Ordinary Portland cement and coarse aggregate with a maximum size of 20 mm were used for the concrete test specimens. The D10 reinforcing-bars (re-bars) of SD 295A type were used. The physical properties of concrete and re-bars are listed in Table 1.

### B. Specimen Sizes

The design of the test specimen was from the Japanese Specifications for Highway Bridge<sup>[6]</sup>. For this present experiment, the half of the model in the Specifications was been designed. Therefore, the sizes of RC slab, arrangement of re-bars and four sides of

simple support for the specimen are shown in Figure 1.

The RC slab specimens had a span of 1200mm between supports and had 135mm overhanging beyond the supports. The total length is 1470mm. First, in the tension side of RC slab, the re-bars were placed at axle-perpendicular direction with 100mm spacing. Moreover, the re-bars were placed again at the axle direction with 100mm spacing. The effective depths ware 105mm and 95mm for axle-perpendicular and axle direction, respectively. In the compression side of the RC slabs, the re-bars were half amounts of tension side.

### C. Test Method (Static-Load Test)

Figure 1 shows the location of the static-load test that was performed by the wheels (diameter 400mm and width 250mm) and stopped at the center of the slab. The load was increased from 0.0kN with 5.0kN increments until the test specimen broke. The deflection and the strain of the concrete and re-bars were measured for each loading.

## 3. Specimen Model and Material Properties and Analysis Procedure for FEM

### A. Specimen Model for FEM

Model was three-dimensional RC slab model with hinge supports on four sides and created by DIANA. Steel Re-bars were included in the modeling, as well. For the mesh type of RC slab, an eight-node quadrilateral isoparametric curved layered shell element was used. Figure 2 shows the model and mesh.

### B. Material Properties for FEM

For the cracking, the Smeared-cracking modeling<sup>[7 and 8]</sup> would be used that deals macroscopically with cracks and reinforcing bars by expressing the average stress and average strain relationships in an element.

## 静荷重が作用する RC 床版の変形

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Table 1 Properties of concrete and steel re-bars

Test specimen	Concrete	Re-bar (SD295A / D10)		
	Compressive strength (N/mm <sup>2</sup> )	Yield strength (N/mm <sup>2</sup> )	Tensile strength (N/mm <sup>2</sup> )	Young's modulus (kN/mm <sup>2</sup> )
RC slab	35	370	511	200

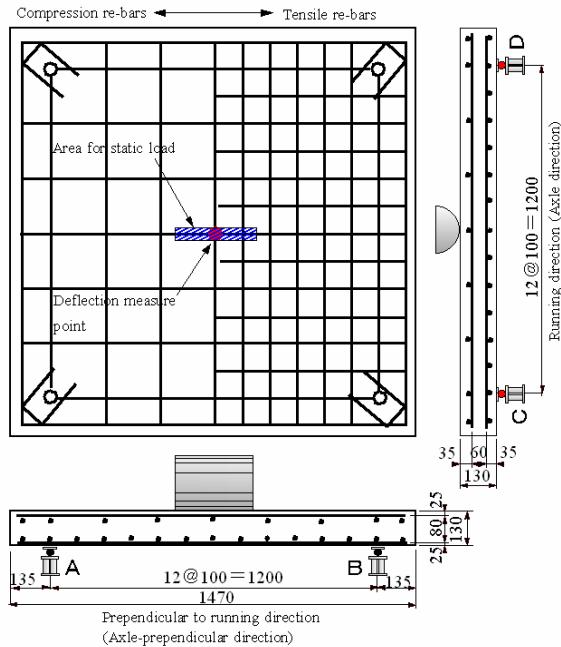


Figure 1 Specimen size and arrangement of steel re-bars

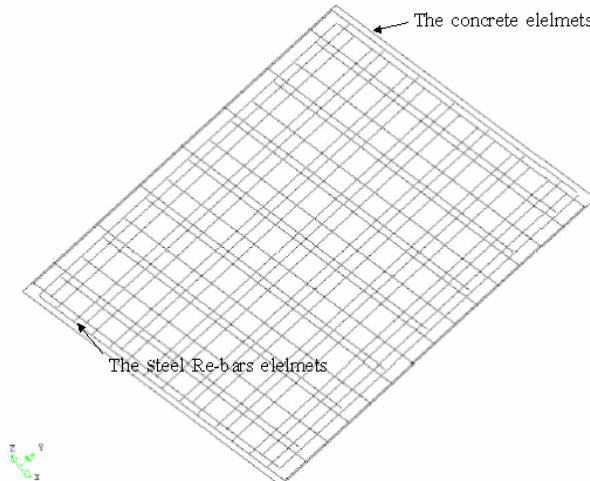


Figure 2 Model and Mesh for RC slab

The plasticity used for the concrete and the steel re-bars was Von Mises [1 and 9] that is a circular cylinder in the principal stress space.

### (1) Concrete parameter

#### (a) Tension strength and tension softening curve

The tension strength was calculated from the compression strength, and Equation 1 was from the ACI code<sup>[10]</sup> that the direct tension strength calculation

with the unit weight of concrete.

For the concrete tension softening curve, the liner tension softening was chosen from DIANA's manual. The ultimate tension strain in the softening curve was used the re-bar's yield strength (Equation 3) because the tension stiffening effect was considered into the tension softening as well. Moreover, in the ACI code<sup>[10]</sup>, there are two approached methods for modeling the tension stiffening as cracking progresses. The first approach is retaining a decreasing concrete modulus of elasticity and leaving the steel modulus unchanged. The second, the steel modulus of elasticity is first to be increased and then gradually decreasing, and the concrete modulus is set to be zero. Therefore, the ultimate strain for the linear tension softening curve will be the changeable value in the modeling. When the steel modulus of elasticity was increasing at first, the yield strain of re-bar would be decreasing ( $\epsilon_s = \sigma_{sy}/E_s$ ). The

$$\sigma_{tk} = g_t [\omega \times \sigma_{ck}]^{0.5} \quad (1)$$

Where,

$\sigma_{tk}$ : Tension strength (N/mm<sup>2</sup>),

$g_t : 0.0069$ ,

$\omega$  : Unit weight of concrete (2350kg/m<sup>3</sup>=23029 N/m<sup>3</sup>),

$\sigma_{ck}$  : Compression strength (N/mm<sup>2</sup>).

$$\sigma_c' = -\frac{\frac{E_{ci}}{E_{cl}} \frac{\epsilon_c'}{\epsilon_{cl}} - \left( \frac{\epsilon_c'}{\epsilon_{cl}} \right)^2}{1 + \left( \frac{E_{ci}}{E_{cl}} - 2 \right) \frac{\epsilon_c'}{\epsilon_{cl}}} \sigma_{ck} \quad (2)$$

where:

$$E_{ci} = E_{co} \left( \frac{\sigma_{ck}}{\sigma_{com}} \right)^{1/3}$$

$E_{ci}$ : the tangent modulus

$E_{co}$ :  $2.15 \times 10^4$  (N/mm<sup>2</sup>);

$\sigma_{co}$ : 10 (N/mm<sup>2</sup>);

$\sigma_c'$ : Stress for compression (N/mm<sup>2</sup>);

$\epsilon_c'$ : Strain for compression;

$\epsilon_{cl}$ : -0.0022;

$E_{cl}$ :  $\sigma_c'/0.0022$ = secant modulus from the origin to the peak compressive stress.

$$\varepsilon_u^{cr} = K_s \frac{\sigma_{sy}}{E_s} \quad (3)$$

where:

$\varepsilon_u^{cr}$ : ultimate strain,

$\sigma_{sy}$ : Steel re-bar's yield ( $N/mm^2$ ),

$E_s$ : Steel re-bar's young's modulus ( $kN/mm^2$ ).

$K_s$ : Coefficient due to the tension stiffening effect

amount of decreasing would be found by try and error method. This changeable value (coefficient  $K_s$ ) would be the main points for this research because this value is going to be effect the results of FEM modeling.

#### (b) Compression strength and compression strain curve

The uniaxial compression test had been done in the laboratory. The compression strengths are list in the Table 1 for the RC slabs. However, the characteristic of the compression curve need to be adopted from the CEB-FIP Mode Code 90 [3]. The equations and compression curve are listed in Equation 2 and Figure 3, respectively. The Young's modulus and yield strength, was 1/3 of compression strength, were found in the Standard Specifications for Concrete Structures [2]. The shear retention should also consider into the cracking modeling, and the full shear retention was used in the modeling of concrete cracking. In the other hand, the shear modulus did not reduce in the DIANA's calculation.

#### (c) Steel Re-Bars for modeling

Steel re-bar's property that used in modeling was bi-linear. In the smeared-cracking modeling, the average stress-strain should be used, and the re-bar was embedded in concrete. After the steel re-bars' yield strength, the slop of  $0.01E$  would be used. Equation 5 is calculation before the steel re-bar yield strength and Figure 4 shows the properties of steel re-bars.

$$\sigma_s = E_s \varepsilon_s \quad (5)$$

Where,

$\sigma_s$  and  $\varepsilon_s$  are the average stress and strain of mild steel re-bar, respectively;  $\sigma_{sy}$  and  $\varepsilon_{sy}$  are re-bar's yield stress and yield strain, respectively;  $E_s$  is the modulus of elasticity for steel re-bar.

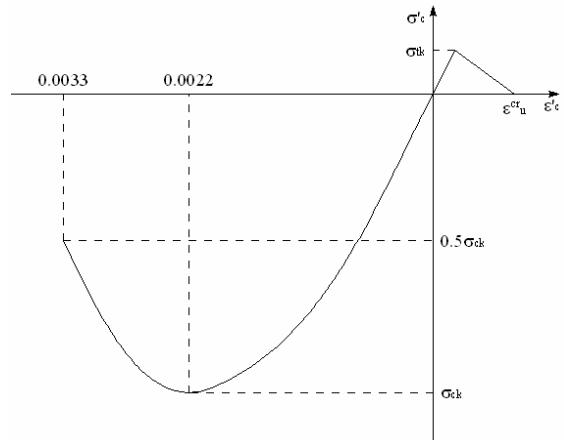


Fig. 3 Concrete's Stress and Strain Relationship

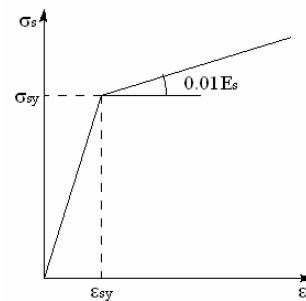


Figure 4 Steel Re-bar's Stress and Strain relationship

### C. Analysis Procedure

The load steps were used for the non-linear analysis calculation, and the increment for the step was 1kN. For the iteration processing, Quasi-Newton method [1], uses the information of previous solution vectors and out-of-balance force vectors during the increment to achieve a better approximation, would be used with displacement and force norms for convergence criteria. One hundred interactions would be used for each load steps, and convergence tolerance would be 0.001.

### 4. Comparing the Test Results with the Modeling

Deflection will be comparing for the experimental results and modeling results that show in Figure 5 for RC slab under the static-load. Finding of the coefficient  $K_s$  was the try and error method. Moreover, form the previous studies [11], the coefficient was 0.1 for modeling the RC beam without the CFS reinforced. In this modeling, the first try for the coefficient  $K_s$  was 0.1, and the modeling results were close to the experimental results. Therefore, the authors concluded that the coefficient  $K_s$  for the ultimate tension strain of the

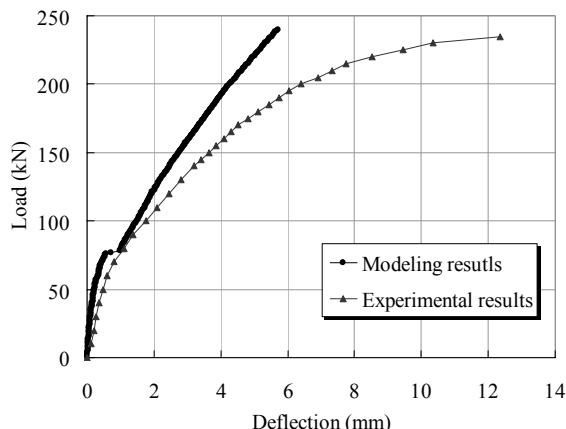


Figure 5 Loads and Deflection Relationship

tension softening curve was 0.1 with one hundred interactions in each load increment.

### (1) Test Results

Under the static-load, the load carrying capacity of RC slabs was 235.0kN. For the axle-perpendicular direction, the steel re-bar reached to its yield strength when the load was between 150.0kN and 155.0kN. For the axle direction, the steel re-bar reached to its yield strength when the load was between 130.0kN and 140.0kN.

### (2) Modeling Results

The calculation was set to stop at 240.0kN. For the concrete in axle and axle-perpendicular directions, the compression strain ( $\varepsilon_{ck}=0.0022$ ) was not reached when the load was 240.0kN by looking at the center point of the model. The concrete compression yield strength was reached between 126.0kN and 127.0kN for axle-perpendicular direction and between 89.0kN and 90.0kN for axle direction. The concrete tension strength was reached between 30.0kN and 31.0kN for axle-perpendicular direction and between 24.0kN and 25.0kN for axle direction.

For the axle-perpendicular direction, the steel re-bar reached to its yield strength when the load was between 116.0kN and 117.0kN. For the axle direction, the steel re-bar reached to its yield strength when the load was between 98.0kN and 99.0kN.

### 5. Conclusion

Looking at the results from the experimental and the modeling, authors made the following conclusion.

- (1) For modeling results to get more close to the experimental results, the input parameters would

be critical and needed to be tried out for each different parameter. The number of interactions would also affect the results because more interaction calculated in each load increment would get the modeling results more close to the experimental results.

- (2) The coefficient  $K_s$ , related to the concrete tension stiffening, was 0.1 for RC slab. It will be one of the controllers for FEM modeling to match with experimental results.
- (3) From the modeling results, the center point of the model had not yet reached to the concrete compression strength when the calculation stopped at 240.0kN.

### References:

- 1) DIANA Finite Element Analysis User's Manual, TNO Building and Construction Research.
- 2) Japan Society of Civil Engineers: Standard Specifications for Concrete Structures - 2002, Structural Performance Verification, (2002).
- 3) CEB-FIP Model Code 90, Thomas Telford.
- 4) Abe, T., Kida, T., Takano, M., Sawano, T., and Kato, K., "Punching shear load-carrying capacity of RC slabs under static and running loads," *Journal of Structural Engineering III*, Vol. 50A, pp.919-926, (2004).
- 5) Nishimura, M., Kida, T., Abe, T., and Hsu, M.C., "Experimental study on punching shear load-carrying capacity of RC slab under biaxial-load," *Cement Science and Concrete Technology*, No.58, pp.642-647, (2004).
- 6) Japan Road Association, *Japanese Specifications for Highway Bridge*, (2003).
- 7) Hajime OKAMURA, Kohichi MAEKAWA, "Nonlinear Analysis and Constitutive Models of Reinforced Concrete", pp. 7~14, (1991).
- 8) W. F. Chen, "Plasticity in Reinforced Concrete", McGraw-Hill Book Company, (1982) pp. 83~85.
- 9) Jan G M. van Mier, "Fracture Processes of Concrete", CRC Press, Inc., Ch. 5, (1997).
- 10) "ACI Manual of Concrete Practice 2001", *ACI International* (2001)
- 11) Ming-Chien Hsu, Study on Fatigue Characteristics for Running Vibration-Load and Rehabilitation Effect of RC Member in Highway Bridge", Ch 6