

# Fracture Analysis of RC Beam under the Static Load

## by using the FEM

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### 1. Introduction

Finite Element Method (FEM) has been widely used in industrial area and in academic area. In the industrial area, the FEM helps designers to design new bridges and buildings in different loading situations. In the academic area, the FEM helps professors and students to understand more about experiments because the FEM shows more detail for the inner part of the experiments. Therefore, authors had used the FEM to model a simple support beam and compared with the experimental results, and the program was called DIANA 8.1.2<sup>[1]</sup>.

that was performed by the wheels (diameter 35.0 cm and width 25.0 cm) stopped in the center of the span, the point where the maximum bending stress occurs. The load was increased from 0.0 kN with 10.0 kN increments until 50kN and changed the increments

**Table 1 Properties of concrete and reinforcements**

Test Specimen	Concrete compressive strength (N/mm <sup>2</sup> )	Reinforcement Bar (SD295A / D16)		
		Yield strength (N/mm <sup>2</sup> )	Tensile strength (N/mm <sup>2</sup> )	Young's modulus (kN/mm <sup>2</sup> )
	39.5	368	568	195.5

### 2. Experimental Material, Specimen Size and Experimental Method<sup>[2]</sup>

#### A. Experimental Material

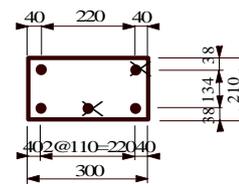
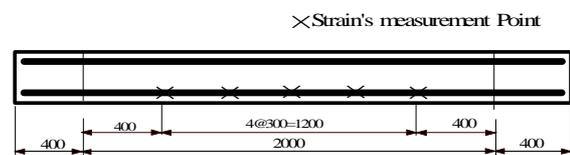
Ordinary Portland cement was used as the concrete for the test specimens, with coarse aggregate of a maximum size of 20 mm. SD 295A and D16 were used as reinforcements. The properties for the concrete and reinforcements are listed in the Table 1.

#### B. Experimental Dimension

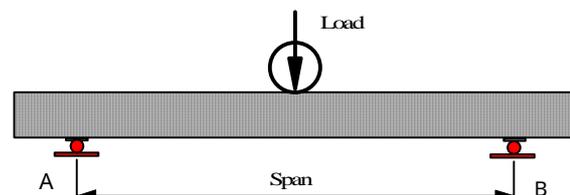
The span of the test specimens was 200 cm and the overhangs were 40 cm on each side that the total length was 280 cm. The width was 30 cm, and the height was 21 cm. Three D16 reinforcements were used on the tension side and two D16 were used on the compression side. The effective depth of the tension reinforcements were 17.2 cm. There was no shear reinforcement in side the test specimens.

#### C. Experimental Method (Bending Test)

Figure 2 shows the bending test using a static load



**Fig. 1 Specimen size and arrangement of reinforcements**

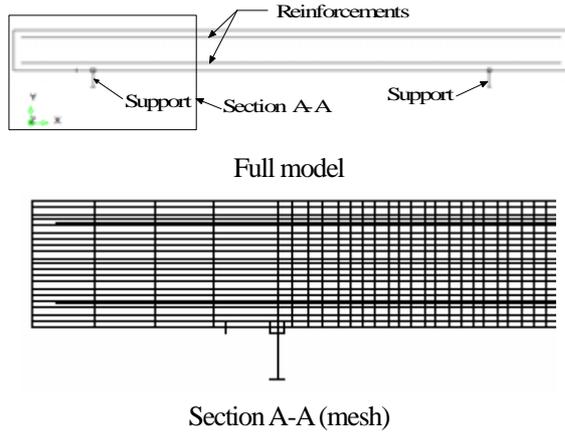


**Fig. 2 Static loading method**

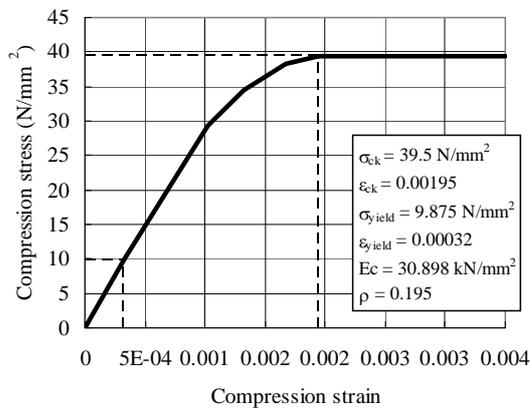
## FEM 解析による静荷重を受ける RC はりの破壊機構

徐 銘謙

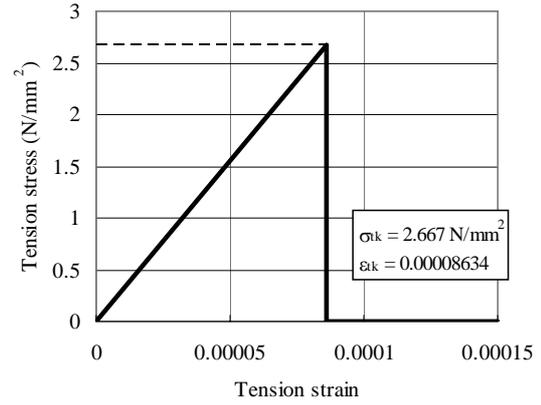
木田 哲量, 小澤 善隆, 澤野 利章, 阿部 忠



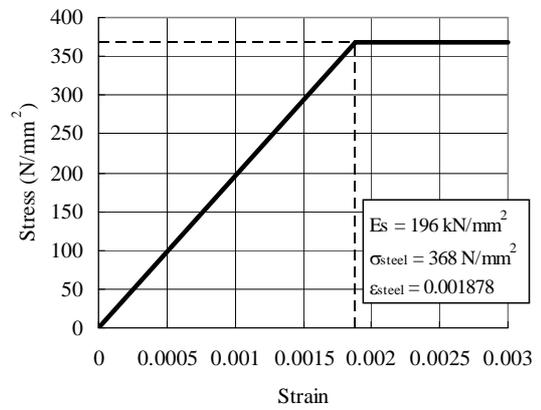
**Fig. 3 Models and meshes**



**Fig.4 Concrete compression strength**



**Fig. 5 Concrete tension strength**



**Fig. 6 Strain-stress for reinforcements**

to 5kN until the test specimen broke. The deflection and the strain of the concrete and reinforcements were measured for each loading.

### 3. Specimen Model and Material Properties and Analysis Procedure for FEM

#### A. Specimen Model for FEM

Model was 2D simple support beam model and created by iDIANA [1]. Reinforcements were included in the modeling, too. For the mesh type, an eight-node quadrilateral isoparametric plane strain element was used. Fig. 3 shows the full model and part of mesh (Section A-A) for the modeling beam.

#### B. Material Properties for FEM

For the cracking, the Smeared-cracking modeling [3 and 5] would be used that deals macroscopically with cracks and reinforcing bars by expressing the average stress and average strain relationships in an element. The Von Mises [1 and 6] plasticity that is a circular cylinder in the principal stress space would be used for

the compression of the concrete and reinforcements.

#### (1) Concrete

The compression strength, young's modules,

$$\sigma_{tk} = 0.23\sigma_{ck}^{2/3} \quad (1)$$

where:

$\sigma_{tk}$ : Tension strength (N/mm<sup>2</sup>),  
 $\sigma_{ck}$ : Compression strength (N/mm<sup>2</sup>).

$$\sigma_{yield} = 0.25\sigma_{ck} \quad (2)$$

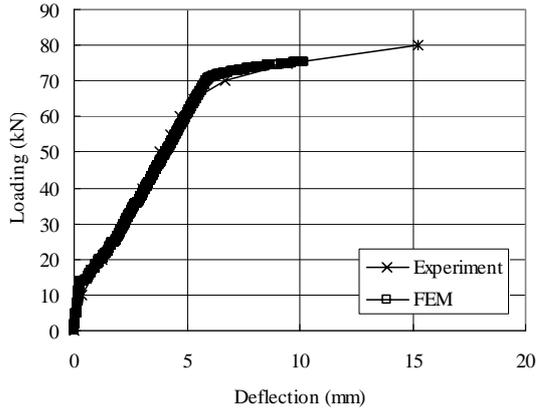
where:

$\sigma_{yield}$ : Compression yield strength (N/mm<sup>2</sup>),  
 $\sigma_{ck}$ : Compression strength (N/mm<sup>2</sup>).

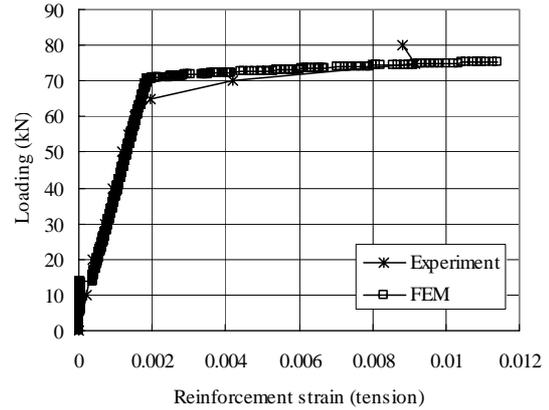
$$\beta = \frac{1}{1 + 4447\varepsilon_{nn}^{cr}} \quad (3)$$

where:

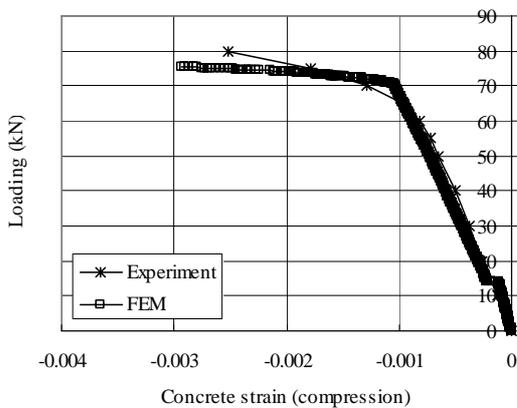
$\beta$ : Shear retention factor,  
 $\varepsilon_{nn}^{cr}$ : Normal cracking strain



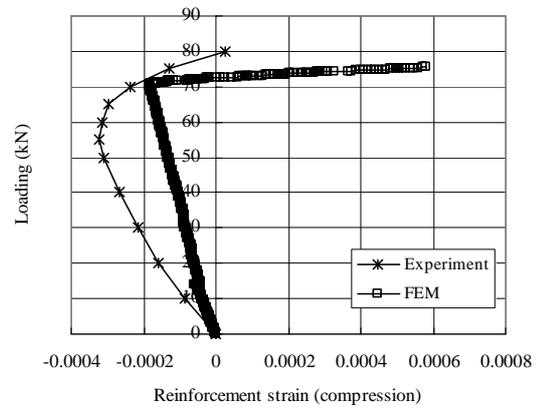
**Fig. 7 Deflection and loading**



**Fig. 9 Reinforcements (tension) and loading**



**Fig. 8 Concrete strain (compression) and loading**



**Fig. 10 Reinforcements (compression) and loading**

poisson ratio, and hardening diagram for the concrete would find in uniaxial compression tests. The tension strength (1) and the compression yield strength (2) might find in the Standard Specifications for Concrete Structures [4]. Since the concrete is brittle material, the brittle cracking modeling would be used for the tension softening. The shear retention factor [1,5 and 6] (3) should be also considered into the cracking modeling and was calculated from the normal cracking strain (or ultimate crack strain.) Fig. 4 and Fig. 5 show the modeling information and the stress-strain relationship for the concrete.

**(2) Reinforcements**

For reinforcements, the stress-strain curve would be bi-linear that shows in Fig. 6.

**C. Analysis Procedure**

The load steps were used for the non-linear analysis calculation, and the increment for the step was 0.05kN until the calculation stopped. For the iterative procedure, the Newton-Raphson [1] would be used, and

the convergence criteria would be force norm and displacement norm.

**4. Comparing the Experiment results with the Modeling**

The results for the experiment would be three test specimens' average that would compare with the FEM results. During the experiment, the deflection of the test specimen, the concrete strains for compression side and the tension strains and the compression strains for the reinforcements were measured at the center of the span. These results show in Fig. 7, Fig. 8, Fig. 9 and Fig. 10 with the FEM results, and some descriptions are listed below

**(1) For the experiment**

The test specimens failed around 80kN. The reinforcements reach its yield point around 65kN to 70kN. The compression reinforcements started to turn into tension when the loading was around 55kN. The initial cracks started between 10kN to 20kN.

## (2) For the FEM

The model failed at 75.4kN. The tension reinforcements reached to its yield point at 70.5kN. The compression reinforcements started to turn into tension when the loading was 70.5kN. The initial cracks started after 13.9kN that the concrete reached to its tension strength. Concrete reached its maximum compressive strength when the loading was 74kN

Looking at the results above, both results are similar but not exactly the same because the information for the material input data will effect the calculation of the FEM, and the FEM's results show more details about the materials. That is why making the material properties for the input data are very important. Also, there were only three test specimens for the experiments, and if there were more test specimens, the average results were be better. The set up of the experiments would affect the results as well.

In the results for the FEM, there were three factors in the input file that were effect the results after the initial cracks started to happen, and they were the shear retention factor, the plasticity and the property of reinforcements.

The shear retention factor ( $\beta$ ) is related to the cracks opening and will reduce the shear modulus of the elasticity. The value for  $\beta$  is between zero and one. When the value of  $\beta$  is zero, a crack is open. When the value of  $\beta$  is one, crack is closed that implies no aggregate interlocking for an open crack and a perfect healing for a closed crack. It could assume that a crack closes when the direct strain across the crack becomes compressive<sup>[5]</sup>.

The Plasticity is specified as yielding or hardening and can observe the permanent deformations or irreversible deformation in the structure<sup>[1]</sup>. These deformations can be related to processes inside the material like concrete.

If using tri-linear property pattern or complete property pattern for the reinforcements besides bi-linear property pattern, the results for the FEM would be different after the yield (70.5kN) of the reinforcements. If we used the tri-linear property pattern, the results after the yield would follow and similar to the tri-linear property pattern before the specimen failed in the FEM. The results for the complete property pattern would be

the same. However, the results for the reinforcements were similar to the bi-linear property pattern in the experiments. That was why the bi-linear property pattern had been chosen.

## 5. Conclusion

- (1) The reinforcements would reach its yield strength before the concrete reached its maximum compressive Strength.
- (2) For getting the similar results as experiments, knowing the material properties and having the correct input data would be the key factor for the modeling.
- (3) After the initial cracks in the RC beam, the shear retention factor for the cracking, the plasticity and the reinforcements' property were effect the FEM modeling.
- (4) The FEM analysis helps to understand the experimental more details and could predict the experimental before hand.

## References:

- 1) DIANA Finite Element Analysis User's Manual, TNO Building and Construction Research., (2000).
- 2) Tadashi ABE, Tetsukazu KIDA, Toshiaki SAWANO, Masaaki HOSHINO, and Kiyoshi KATO, "Flexural Load-Carrying Capacity and Failure Mechanism of RC Beams with Low Effective Depth under Running Wheel-Load.", Materials Science Research International, Vol.7, No.3, (2001) pp. 186~193.
- 3) Hajime OKAMURA, Kohichi MAEKAWA, "Nonlinear Analysis and Constitutive Models of Reinforced Concrete", (1991) pp. 7~14.
- 4) Japan Society of Civil Engineers: Standard Specifications for Concrete Structures - 2002, Structural Performance Verification, (2002).
- 5) W. F. Chen, "Plasticity in Reinforced Concrete", McGraw-Hill Book Company, (1982) pp. 83~85.
- 6) Jan G. M. van Mier, "Fracture Processes of Concrete", CRC Press, Inc., (1997) Ch. 5